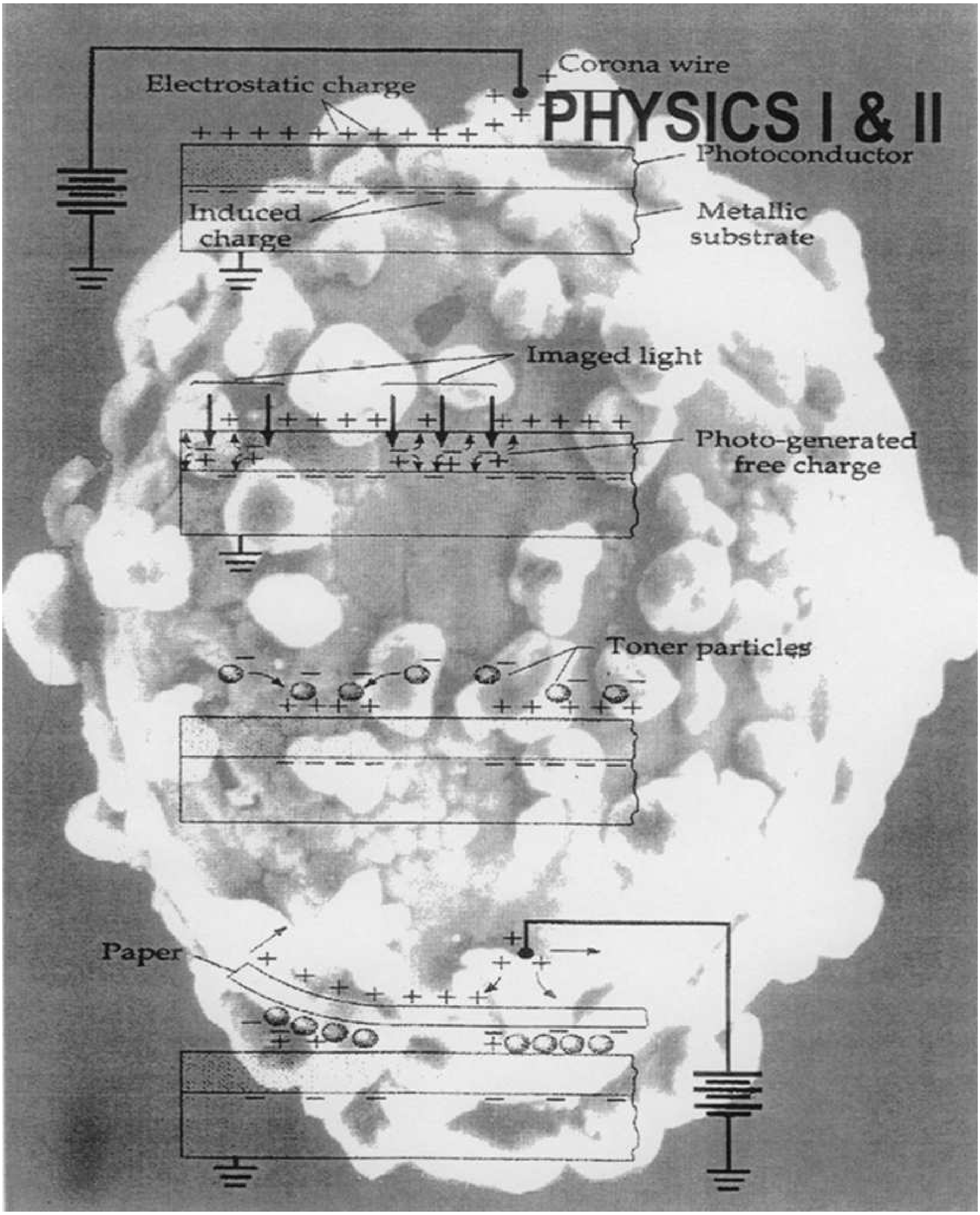

PHYSICS I & II
Corona wire
Electrostatic charge
Photoconductor
Induced charge
Metallic substrate
Imaged light
Photo-generated free charge
Toner particles
Paper

ELECTROSTATICS

Charge – A Definition

Protons and electrons have a quality referred to as electric charge that confers upon them an attractive force. Electric charge can be positive and negative. A positive particle will attract a negative particle, and particles of identical charge will repel each other. Protons are positively charged and electrons are negatively charged. Neutrons are electrically neutral and, thus, have no charge. Charge is expressed in Coulombs (C).

The number of electrons in an electrically neutral body is equal to the number of positive charges, usually protons.

Charge is generally measured in coulombs

A coulomb is the quantity of charge of a standard number of elementary charges.

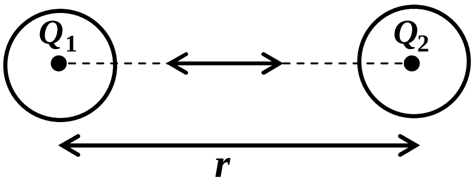
6.25×10^{18} elementary charges = 1 coulomb

This relationship can be expressed per elementary charge.

1 elementary charge = 1.6×10^{-19} coulombs

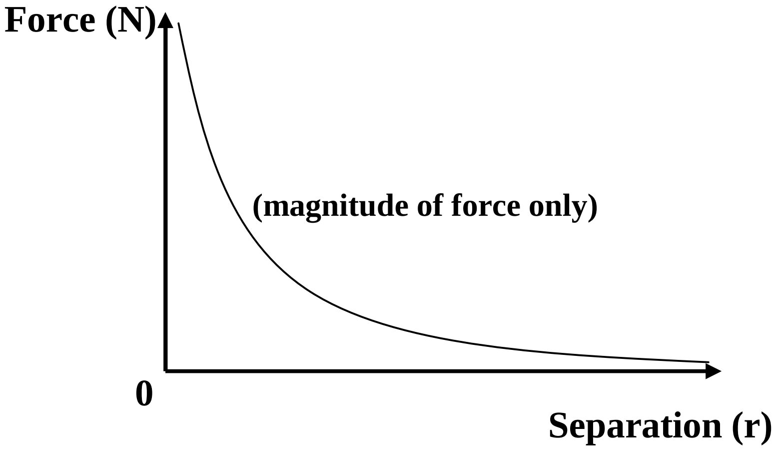
Coulomb's Law of Electric Force

The force of attraction or repulsion between two charges Q_1 coulombs and Q_2 coulombs positioned a distance r metres apart is proportional to the product of the charges and inversely proportional to the square of the distance between the two charges.



$$F \propto \frac{Q_1 Q_2}{r^2}$$

If Q_1 and Q_2 both have the same sign, then F will be positive and will represent a force of repulsion. If the charges have opposite signs, then F will be negative and will represent a force of attraction.



The value of the proportionality constant, k, depends on the material between the charged particles. In a vacuum, $k = 9 \times 10^9\ \mathrm{Nm^2/C^2}$. Recollect that the value of the universal gravitational constant, G, is $6.7 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$. Since the value of k is twenty orders of magnitude greater than G, the relative strength of the electrostatic force is clearly larger than the gravitational force.

$$F = \frac{kQ_1Q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon}$$

ε, called the **permittivity** of the material, is constant for a particular material.

A charge of 1 coulomb is possessed by a point if an equal charge placed 1 metre away from it in a vacuum experiences a force of $\frac{1}{4\pi\varepsilon}$ newton.

The electrostatic force obeys the same inverse-square relationship with distance as the gravitational force. However, while the gravitational force is proportional to mass and always attractive, the electrostatic force is proportional to charge and can be either attractive or repulsive. And as already noted, for both macroscopic objects and subatomic particles, the electrostatic force is much stronger than the gravitational force. Indeed, it is the electrostatic and not the gravitational force that binds atoms to one another.

Comprehension Question

A sphere with a charge of $+Q$ is fastened in position. A smaller sphere of charge $+q$ is positioned in proximity to the larger sphere and released from rest. The small sphere will travel away from the large sphere with

A decreasing velocity and diminishing acceleration.

B decreasing velocity and increasing acceleration.

C decreasing velocity and constant acceleration.

D increasing velocity and diminishing acceleration.

Answer

D is correct. As distance *r* increases (i.e. as the small sphere is pushed away), acceleration *a* decreases. However, since *a* is always positive, the small sphere's velocity, *v*, is always increasing.

Electric Field

An electric field is the space around an electrically charged body or source charge in which it can exert a force on another charged body.

The **electric field strength** E around a source charge Q (which causes a force on a small positive charge q when it enters the field) is given by $E = F/q$, where E is the symbol for electric field (around any source charge Q) with units of N C^{-1}, at a distance of r (m) from a small positive charge q with a force of F (N) between the two charges. The electric field direction (depicted by electric field lines) is defined to be the way in which the small positive charge q will move and the field strength varies inversely with distance squared. Since the force on the charge q has a strength of kqQ/r^2, when divided by q, the expression for the strength of the electric field created by a source charge of magnitude Q results: $E = k(Q/r^2)$.

$$\boxed{E = \frac{F}{q}} \quad \text{and since} \quad \left(F = \frac{kQq}{r^2}\right) \quad \therefore \quad \boxed{E = \frac{kQ}{r^2}}$$

If the source charge Q is positive, the electric field vectors point away from it; if the source charge Q is negative, then the field vectors point toward it. Because the force decreases as distance increases from the charge (as $1/r^2$), the electric field decreases proportionally. Note that the electric field vectors further away from the source charge Q are less densely arranged than those that are closer in order to graphically depict a diminishing electric field strength. While depicted in two dimensions, E is really a three dimensional phenomena.

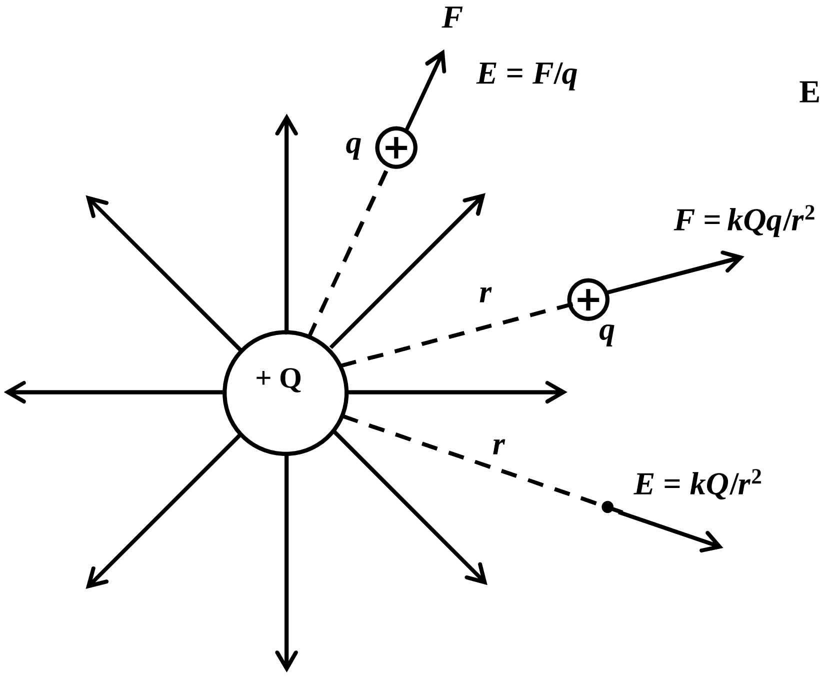
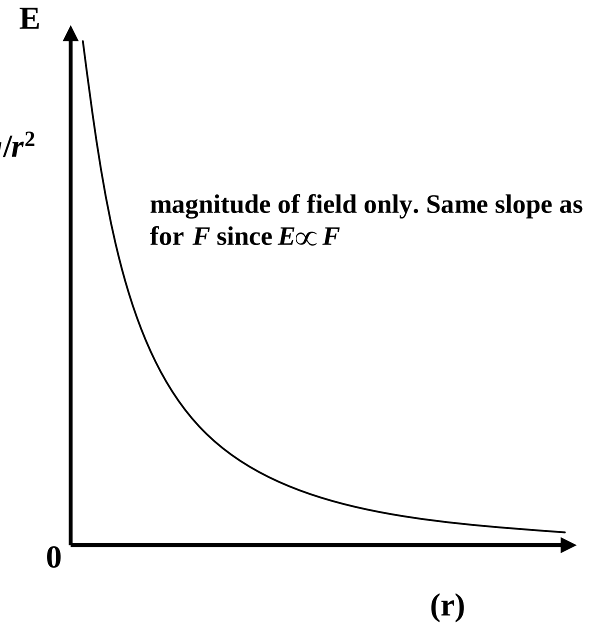


Diagram of electric field lines surrounding a source charge of +Q imposing a force on a small positive charge q.



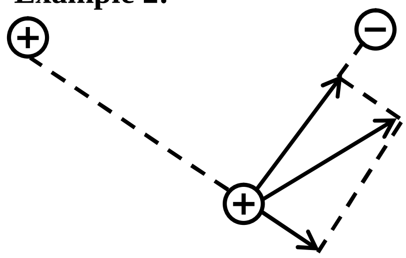
Electric field around and isolated charge versus distance

Example

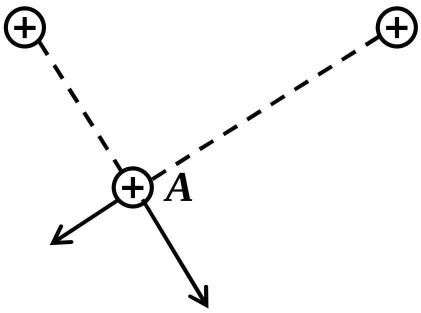
To find the electric force at point A.

Example 1:

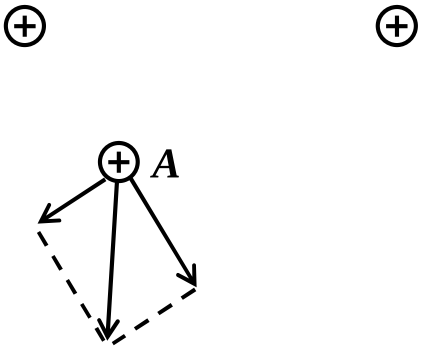
Example 2:



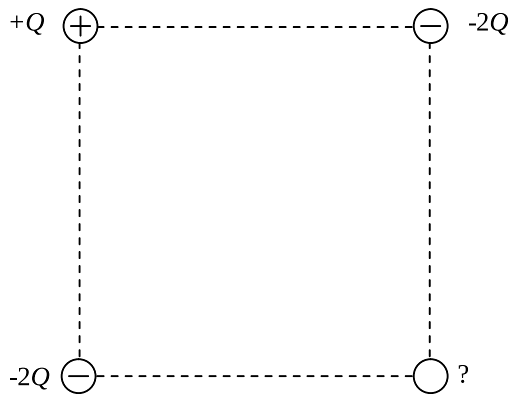
Consider the individual attractions or repulsions on a small positive charge placed at *A*.



Sum the vectors,



Comprehension Question



The illustration demonstrates four point charges fastened in place at the corners of a square. What quantity of charge must be present at the bottom right corner so the electric field at the centre of the square is zero?

A $+Q$

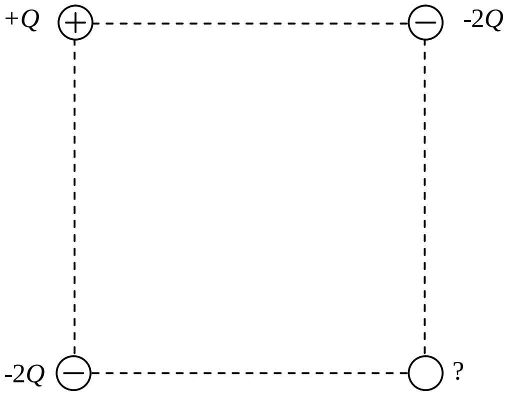
B $+Q\sqrt{2}$

C $+2Q$

D $+3Q$

Answer

A is correct. The individual electric field vectors at the centre of the square originating from the two identical negative source charges cancel each other out. The individual electric field vector at the centre of the square due to the bottom right mystery source charge must cancel out the individual electric field vector at the centre of the square due to the upper left source charge. Since the upper left source charge is $+Q$ the bottom right source charge must also be $+Q$.



Electric Potential

Electric potential (or voltage) is defined as the potential energy per unit of positive test charge that would exist at a point in space if a unit of positive test charge was placed there.

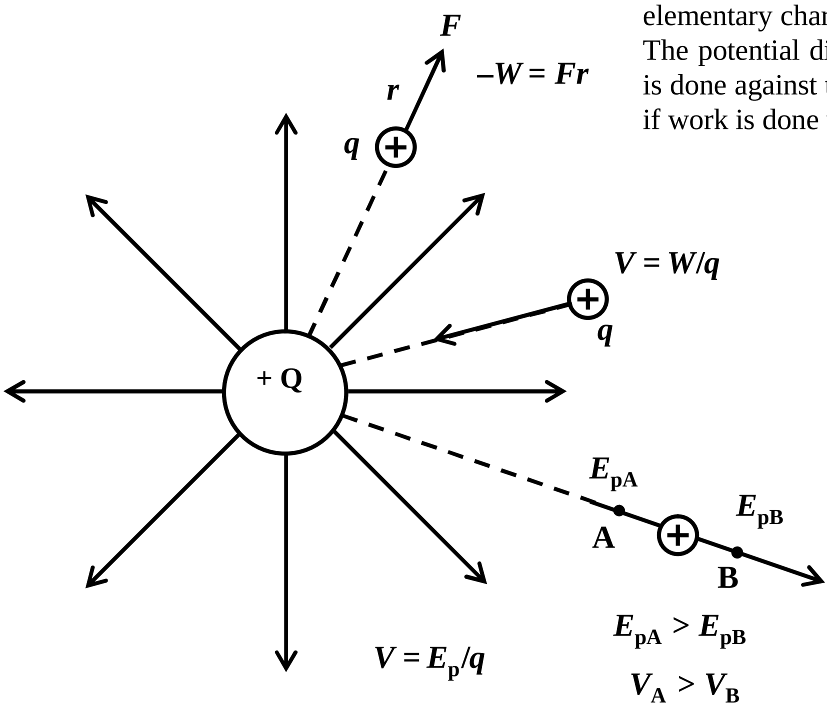
Potential energy E_p, expressed mechanically, is the product of force F and distance r, which is work W $\therefore$ $E_p = Fr = W$

Electrostatically $E_p = \frac{KQq}{r^2}r$ (using Coulomb's law) $\therefore E_p = \frac{KQq}{r}$ $\therefore \frac{E_p}{q} = \frac{KQ}{r}$

And Potential energy per unit charge is called electric potential, or voltage V.

$$\boxed{V = \frac{KQ}{r}}$$

The absolute electrical potential V at a point is the work W done in moving a unit positive charge q from infinity to that point. In practice it is often more convenient to compare the potential at one point relative to another rather than know its absolute potential. If the potential at point A is V_A and the potential at point B is V_B then the potential difference between A and B is $V_A - V_B$ and represents the difference in the work W done in moving a unit positive charge q from infinity to point A and from infinity to point B. It can be seen that this is the same as the work W which would be required to move a unit positive charge q from point A to point B. This leads to the definition of potential difference: the potential difference between two points is the work W done on a unit positive charge q in moving it from one point to the other. The volt is the SI unit of potential defined as: 1 volt of potential exists at a point if 1 joule of work is performed in moving 1 coulomb of positive charge from infinity to that point. Similarly for potential difference, 1 volt of potential difference exists between two points if 1 joule of work is done in moving 1 coulomb of positive charge from one point to the other. In addition, Potential difference is measured in Joules per elementary charge or in Joules per coulomb (volt.) The potential difference is a potential rise if work is done against the field (+W), and a potential drop if work is done with the field (– W).



$$\boxed{V = \frac{W}{q}}$$

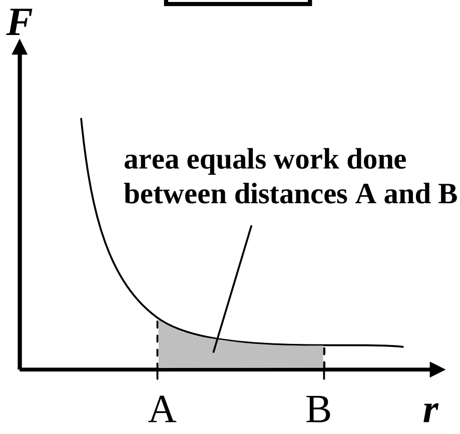


Diagram of energy surrounding a point charge of +Q

Electrostatics: *Quick Questions*

Questions 1 – 3: Charge +q is located a distance r from charge +Q. Each charge is 1 coulomb.

1. The electric field due to charge +Q at a distance r is equal to

A kQ/F **B** kQq/r
C kQ/r **D** kQ/r^2

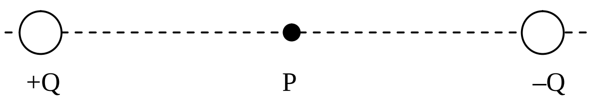
2. If 100 J of work is needed to move +q through a distance r to +Q, what is the potential difference between the two charges? (+q = +1 coulomb)

A 200 V **B** 100 V
C 400 V **D** 50 V

3. If distance r is doubled, then the force that +Q exerts on +q is

A divided by 4 **B** divided by 2
C unchanged **D** multiplied by 2

4.



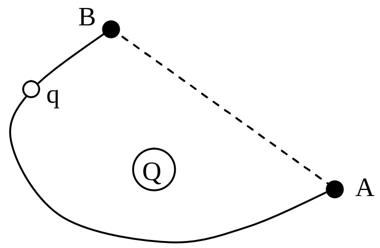
The diagram above illustrates two charges, +Q and –Q. If the negative charge was nonexistent, the electric field at point P due to +Q would have magnitude E. With –Q present, what is the magnitude of the total electric field at P, which occupies the midpoint of the line joining the centres of the charges?

A E/4 **B** E/2
C E **D** 2E

5. Negative charges are accelerated by electric fields toward areas

A of lower electric potential
B of higher electric potential
C where the electric field is zero
D where the electric field is weaker

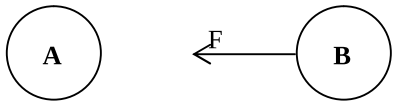
6.



How much work would the electric field, due to stationary charge Q, do as a charge q travels from point A to B along the curved path illustrated? (V_A = 100 V, V_B = 50 V, q = –0.025 C, length of line segment AB = 5cm, length of curved path = 10cm).

A –1 J **B** –1.25 J
C +0.5 J **D** +1 J

Questions 7 – 10:



A metal sphere, A, has a charge of –q coulombs. An identical metal sphere, B, has a charge of +2q coulombs. The magnitude of the electrical force on B due to A is F newtons.

7. What is the magnitude of the electric force on A due to B?

A ½ F **B** F
C 2F **D** 4F

8. If the distance between the centres of the spheres is quartered, what is the magnitude of the force on B due to A?

A ½ F **B** 4F
C 8F **D** 16F

9. If an electron were placed halfway between A and B, the resultant electric force on the electron would be

A toward A **B** toward B
C zero **D** out of the page

10. If A and B are joined by a wire conductor with negligible surface area compared to the spheres, charge will flow through the connecting wire until the charge on B becomes

A 0 C **B** +q/2 C
C +q C **D** –q/4 C

1: D, 2: B, 3: A, 4: D, 5: B, 6: B, 7: B, 8: D, 9: B, 10: B

Electrostatics: *Quick Solutions*

1. **(D)** The force between the centres of two charges is:

$F = kQq/r^2$

The field created by one charge is the electric force it exerts on another unit charge:

$E_Q = F/q = kQ/r^2$ (q also creates a field equal to kq/r^2)

2. **(B)** The energy per coulomb or work needed to move +q a distance r in the field created by +Q is the potential difference. This is given by W = qV. If 100 J are needed to move +q, which equals 1 coulomb, then there is a potential difference of 100 volts.

3. **(A)** Electrical force and r^2 are inversely proportional, where $F = kQq/r^2$. If r is multiplied by two, then force is divided by four.

4. **(D)** P is at an equal distance from both charges and the magnitudes of the charges is the same. Hence the magnitude of the electric field at P due to +Q and the magnitude of the electric field at P due to –Q are also the same. The electric field vector at P due to +Q is directed away from +Q and the electric field vector at P due to –Q directed toward –Q. Since these vectors are oriented in the same direction, the net electric field at P is (E to the right) + (E to the right) = (2E to the right).

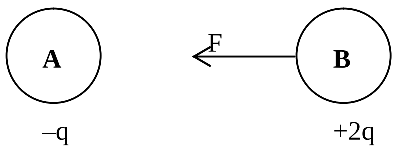
5. **(B)** The equation $\Delta V = -W/q$ refers to the energy per coulomb or work required to move q through a potential difference ΔV. If an electric field does positive work on a negative charge by accelerating it, then $W > 0$. If $q < 0$, then $-W/q$ is positive. Therefore, ΔV is positive, suggesting that V increases.

6. **(B)** By definition $W = -q\Delta V$, which gives

$W = -q(V_B - V_A) = -(-0.025\ \text{C})(50\ \text{V} - 100\ \text{V}) = -1.25\ \text{J}$

Notice that neither the length of the segment AB nor that of the curved path from A to B is relevant.

7. **(B)**



Newton's third law holds that if A exerts a force on B, then B exerts an equal but oppositely directed force back on A. Hence, the magnitude of the force on A due to B is also F newtons.

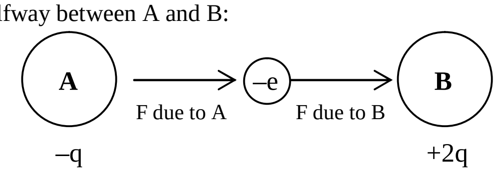
8. **(D)** The force is inversely proportional to r^2. So:

$F \propto 1/r^2$

If $r_2 = 1/4\ r_1$, then

$F_{old} \propto 1/r_1^2$ and $F_{new} \propto 1/r_2^2 = 1/(1/4\ r_1)^2 = 1/(1/16\ r_1^2) = 16\ (1/r_1^2)$. Thus the new force is 16 times greater.

9. **(B)** If an electron with its negative charge (–e) was placed halfway between A and B:



The electron would be attracted by B (opposite charges attract) and repelled by A (like charges repel). Hence, the resultant force would be toward B and away from A.

10. **(B)** Charge will flow through the conducting wire until A and B are equally charged. Specifically, +q coulombs from B will neutralise –q coulombs on A, leaving the remaining +q coulombs on B to be equally distributed between A and B. Ultimately, A and B will each have +q/2 coulombs.